**THREE PHASE COORDINATE MODEL**

The induction machine may be viewed as a system of electric and magnetic circuits which are coupled magnetically and/or electrically. An assembly of resistances, self inductances, and mutual inductances is thus obtained. Let us first deal with the inductance matrix. A symmetrical (healthy) cage may be replaced by a wound three-phase rotor. [2] Consequently, the IM is represented by six circuits, (phases) (Figure 14.1). Each of them is characterized by a self inductance and 5 mutual inductances. The stator and rotor phase self inductances do not depend on rotor position if slot openings are neglected. Also, mutual inductances between stator phases and rotor phases, respectively, do not depend on rotor position. A sinusoidal distribution of windings is assumed. Finally, stator/rotor phase mutual inductances depend on rotor position 



 ; 

; ;



Assuming a sinusoidal distribution of windings, it may be easily shown that



Reducing the rotor to stator is useful especially for cage rotor IMs; no access to rotor variables is available. In this case, the mutual inductance becomes equal to self inductance  and the rotor self inductance equal to the stator self inductance   . To conserve the fluxes and losses with stator reduced variables,

;





The expressions of rotor resistance, leakage inductance, both reduced to the stator for both cage and wound rotors are given in Chapter 6. The same is true for, of the stator. The magnetization self inductancehas been calculated in Chapter 5.

Now the matrix form of phase coordinate (variable) model is











 is

With (13.7) , (13.8) becomes



Multiplying (13.10) by we get



The first term represents the winding losses, the second, the stored magnetic energy variation, and the third, the electromagnetic power.



The electromagnetic torque  is



The motion equation is



An 8th order nonlinear model with time-variable coefficients (inductances) has been obtained, even with core loss neglected. Numerical methods are required to solve it, but the computation time is prohibitive. Consequently, the phase coordinate model is to be used only for special cases as the inductance and resistance matrix may be assigned any values and rotor position dependencies. The complex or space variable model is now introduced to get rid of rotor position dependence of parameters.

**THE COMPLEX VARIABLE MODEL**

Let us use the following notations:





Based on the inductance matrix, expression (13.9), the stator phase a and rotor phase  flux linkages  and  are





We may now introduce the following complex variables as space phasors:

[1]





Also,





In symmetric steady-state and transient regimes,



With the above definitions,  and  become





Similar expressions may be derived for phases  and. After adding them together, using the complex variable definitions (13.18) and (13.19) for flux linkages and voltages, also, we obtain





Where



;

In the above equations, stator variables are still given in stator coordinates and rotor variables in rotor coordinates.

Making use of a rotation of complex variables by the general angle  in the stator and  –  in the rotor, we obtain all variables in a unique reference rotating at electrical speed ,



 

With these new variables Equations (13.25) become





For convenience, the superscript b was dropped in (13.30). The electromagnetic torque is related to motion-induced voltage in (13.30).



Adding the equations of motion, the complete complex variable (space phasor) model of IM is obtained.



The complex variables may be decomposed in plane along two orthogonal d and q axes rotating at speed to obtain the d–q (Park) model. [2]





With (13.33), the voltage Equations (13.30) become











Also from (13.27) with (13.19), the Park transformation for stator P () is derived.





The inverse Park transformation is



A similar transformation is valid for the rotor but with  –  instead of.

It may be easily proved that the homopolar (real) variables ,do not interface in energy conversion





And are the homopolar inductances of stator and rotor. Their values are equal or lower (for chorded coil windings) to the respective leakage inductances  and .